A Simple Question with an Interesting Answer

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Here's an inauspicious little question that most people are sure to get wrong the first time they hear it. Given two identical containers that are placed under two different faucets that emit *exactly* the same amount of water every second, which container will fill up first?

The Hasty Answer

The "obvious" answer, of course, is that both faucets will fill up at exactly the same time. Since the water level is found by dividing the volume of water in the container by its cross-sectional area, it would appear that

$$h(t) = \frac{Qt}{A} \qquad (1)$$

where t is the time, Q is the flow rate of water (volume per unit time), and A is the cross-sectional area of the container. Clearly, if Q and A are the same for each container, then the water levels will also be the same, right? Well, that would be true if the water level were really given by Eq. (1), but it's not.

The More Refined Answer

The problem with Eq. (1) is that it assumes that Qt is the volume of water in the container at time t. However, what it *really* represents is the amount of water that has left the faucet at time t, not all of which will have made its way into the container. In fact, at an arbitrary time, there will be both water in the container and water between the faucet and the container. Therefore, even though the two faucets are emitting the same amount of water every second, it is possible to have *more* water in one container if it has *less* water between the faucet and the container.

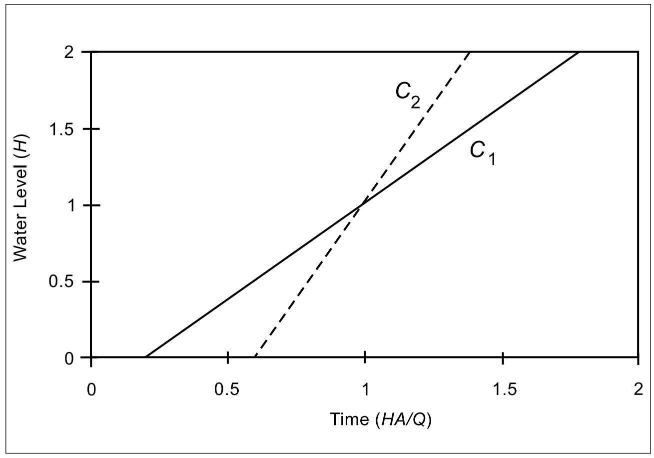

Fig. 1. Graph of water level for two identical containers as a function of time. The faucet filling container C_1 has a cross-sectional area of $a_1 = 0.2A$; the one filling C_2 has an area of $a_2 = 0.6A$. The flow rate, Q, is the same for both faucets.

To simplify the analysis, we will neglect the acceleration due to gravity, which lessens (but does not eliminate) the effect we are after. Again, let A be the cross-sectional area of the container to be filled, and a be the cross-sectional area of the faucet. Then, if the flow rate is given by Q, the velocity of the water as it leaves the faucet must be equal to Q/a. Therefore, we can immediately see that the smaller the faucet area, the faster the water will be traveling. If the faucet is at a height H above the bottom of the container, then it will take a time $t_s = H/v = Ha/Q$ before the container even begins to fill with water. Thus, the smaller the faucet area, the less time it takes to start filling the container. Furthermore, the "cylinder of water" between the faucet and the container will contain *less* water if the faucet area is smaller. This means that a higher percentage of the water that has left the faucet will have gone into filling the container, resulting in a higher water level.

To see this mathematically, consider a time $t > t_s$ at which the container is filled to some level h. We know that the total volume of water that has left the faucet is given by Qt, and this must be divided between the volume in the container, Ah, and the volume in the "cylinder" given by $a(H - h)$. Equating these and solving for the water level yields

$$h(t) = \frac{Qt - aH}{A - a} \qquad (2)$$

For times earlier than this, the water level is of course, equal to zero. Equation (2) shows that the water level does indeed increase linearly with time, but the rate of increase depends on the faucet diameter. This behavior is summarized in Fig. 1, which shows how the water level rises in two identical containers, C_1 and C_2, placed under two different faucets, f_1 and f_2, having respective diameters $d_1 < d_2$. Because f_1 has a smaller diameter, container C_1 begins filling first and therefore has a higher water level initially. However, since the water level is *changing* more rapidly in C_2, it is actually filling at a faster rate. So, while C_2 is losing the race initially, it is gaining on, and will at some point, overtake C_1.

The point at which C_2 will overtake C_1 occurs when $h_1(t) = h_2(t)$, which happens at $t_c = AH/Q$. At this time, both water levels have reached a height H, and all of the water that has left the faucets has gone into filling the containers (assuming, of course, that the height of the containers is at least H); that is, there is no more "cylinder" of water between the faucet and the container. Interestingly, from this point on, the height of the water level is still given by Eq. (2) because the "cylinder" of water, given by $a(H - h)$, now represents the "cylinder" of space above height H, from which water from the faucet *cannot* go. So, to answer the initial question posed, if the top of the container is below the faucet (which is typically the case), then the container under the smaller-diameter faucet will fill up faster. But, if the top of the container is above the faucet, then the larger-diameter faucet will fill the container the fastest.

Is This an Observable Effect?

Although I have not tried to observe this phenomenon, I have given a little thought as to what might be the best way to go about it. Clearly, the larger the difference in diameter between the two faucets, the better. But more importantly, the diameter of the containers should not be too much bigger than the diameter of the larger faucet. It might be possible to use two identical syringes with different sized nozzles for the "faucets." Then, pushing down on both pistons simultaneously would produce the same flow rate from each of these faucets. The containers would need to be tall, thin cylinders, not too much larger in diameter than the larger of the syringe nozzles. In addition, it would be easiest to observe this effect if gravity were not speeding up the liquid as it left the faucet and if you could reduce the splashing of the liquid when it lands in the container. A viscous, honey-like fluid should help to reduce both of these effects. Of course, it may not be easy to force a viscous fluid through a syringe with a small opening, but it seems as if it should be feasible. By videotaping the experiment and using a video-analysis software program, a graph such as Fig. 1 should be easy to produce.

et cetera...

It Is Necessary to Forget

"A major obligation of memory is to forget. If we retained a memory of all the stimuli raining in on our senses every second we would be unable to process the data fast enough to react adaptively to new challenges. To prevent an information overload, our senses and our minds have been designed (by evolution) to slough off all traces of most of the stimuli. Selective censorship is risky, but no censorship at all would be fatal. We are the selected products of a system of censorship that survives. It is not perfect, but it works." [1]

1. Garrett Hardin, *Promethean Ethics: Living with Death, Competition, and Triage* (University of Washington Press, Seattle, 1980) p. 4.

I Want to See This

In a shopping catalog in the seat pocket of the airliner I saw a description of a 1600-watt electric hair dryer. The description said, "There's a lot of power stored in this sleek dryer." [1]

1. *Continental Sky Mall,* Spring 1994, p. 113.

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